Unit 13b: Further Trigonometry

Trigonometry with bounds

The upper bound of a multiplication is always the two upper bounds multiplied together

The lower bound of a multiplication is always the two lower bounds multiplied together

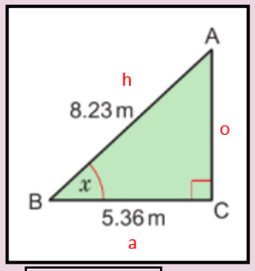
The upper bound of a fraction is always

$$\frac{Upper\ bound\ of\ the\ numerator}{Lower\ Bound\ of\ the\ denominator}$$

The lower bound of a fraction is always

$$\frac{Lower\ bound\ of\ the\ numerator}{Upper\ Bound\ of\ the\ denominator}$$

In this diagram, the measurements are correct to 3 significant figures.
a Find the upper and lower bounds for the value of x, to 3 decimal places.
b Give the value of x to a suitable level of accuracy.



UB of h = 8.235
LB of h = 8.225

UB of a = 5.365
LB of a = 5.355

$$cosx_{ub} = \frac{5.365}{8.225}$$

$$x_{ub} = \cos^{-1}\frac{5.365}{8.225} = 49.438°$$

$$cosx_{lb} = \frac{5.355}{8.235}$$

$$x_{lb} = \cos^{-1}\frac{5.355}{8.235} = 49.286°$$

$$x = 49°\ (to\ the\ nearest\ degree)$$

Steps:

1) Find the upper bound And lower bound of the sides

2) Find the upper and lower Value of x using trigonometry

3) Round both values to the Nearest degree to find a Good estimate for x

Finding a missing side

Step 1: Label your sides and angles

Step 2: Substitute known values into the formula

Step 3: Rearrange the formula to find the missing side

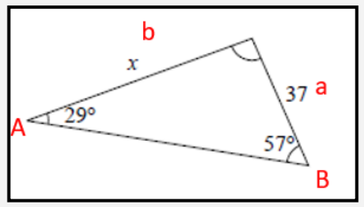
Find the length of side x.

$$\frac{37}{sin29} = \frac{x}{sin57}$$

$$\frac{37 \times sin57}{sin29} = x$$

$$x = 64.004°$$

$$x = 64°$$



Sine Rule

The Sine and Cosine Rules are used for finding missing sides and angles on non right angled triangles.

Finding a missing angle

$$\frac{SinA}{a} = \frac{SinB}{b} = \frac{SinC}{c}$$

$$\frac{SinA}{a} = \frac{SinB}{b}$$

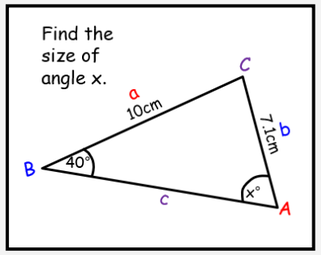
$$\frac{SinA}{10} = \frac{Sin40}{7.1}$$

$$SinA = \frac{Sin40}{7.1} \times 10$$

$$A = \sin^{-1}\frac{Sin40}{7.1} \times 10$$

A = 64.9°

Find the size of angle x.



The formula for the sine rule is

$$\frac{a}{SinA} = \frac{b}{SinB} = \frac{c}{SinC}$$

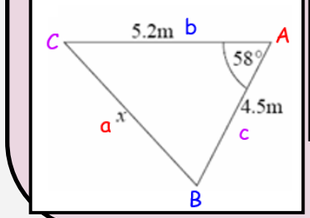
Cosine Rule: Finding a missing side

Can be used to find missing sides or angles. MUST have 2 sides and 1 angle

Formula for missing side

$$a^2 = b^2 + c^2 - 2bcCosA$$

Find the length of side x.



$x^2 = 5.2^2 + 4.5^2 - (2 \times 5.2 \times 4.5 \times Cos58)$

$x^2 = 5.2^2 + 4.5^2 - (24.8)$

$x^2 = 27.04 + 20.25 - (24.8)$

$x^2 = 47.29 - (24.8)$

$x^2 = 22.49$ — Square root both sides

$x = 4.74m$

Cosine Rule: Finding a missing angle

Formula for missing angle

$$CosA = \frac{b^2 + c^2 - a^2}{2bc}$$

To find a missing angle…

MUST have all 3 sides given

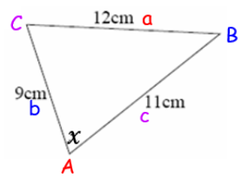
$$Cosx = \frac{9^2 + 11^2 - 12^2}{2 \times 9 \times 11}$$

$$Cosx = \frac{58}{198}$$

$$x = \cos^{-1}\frac{58}{198}$$

$$x = 72.97°$$

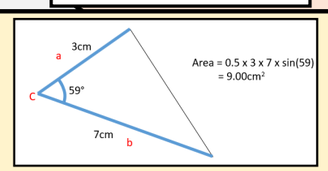
Find the size of angle x.



Area of a triangle

$$Area = \frac{1}{2}a\ b\ \sin(C)$$

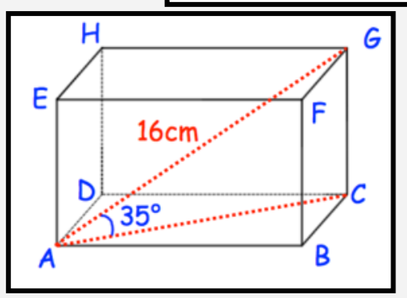
Where C is the angle wedged between two sides a and b.



Cosine Rule

Length AG = 16cm
Angle CAG is 35°

Work out the length of EG.



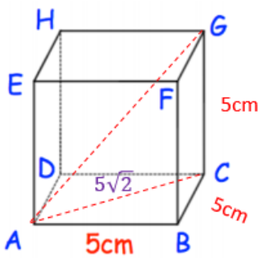
$$\cos(CAG) = \frac{a}{h}$$

$$\cos(35) = \frac{EG}{16}$$

$$16 \times \cos(35) = EG$$

EG = 13.1cm

1. Shown is a cube with side length 5cm.



Calculate angle CAG.

$$AC^2 = \sqrt{5^2 + 5^2}$$

$$AC^2 = \sqrt{50}$$

$$AC = 5\sqrt{2}$$

$$\tan(CAG) = \frac{5}{5\sqrt{2}}$$

$$CAG = \tan^{-1}\frac{5}{5\sqrt{2}}$$

$$CAG = 35.3°$$

Trigonometry in 3D