Unit 17: Further Algebra

Algebraic Fractions

When we multiply fractions we multiply the numerators together and the denominators together

If the fraction shares common factors then we can cross cancel by dividing the factors out before we multiply

$$\frac{7}{11} \times \frac{55}{7} = \frac{1}{1} \times \frac{5}{1} = 5$$

$$\frac{a^2}{2} \times \frac{4}{a} = \frac{a}{1} \times \frac{2}{1} = \frac{2a}{1} = 2a$$

$$\frac{3(c+2)}{8c} \times \frac{2c(c-1)}{c+2} = \frac{3}{4} \times \frac{c-1}{1} = \frac{3(c-1)}{4}$$

When we divide fractions we find the reciprocal of the second fraction and then multiply the fractions together.

$$\frac{6x}{2y} \div \frac{4y}{5} = \frac{6x}{2y} \times \frac{5}{4y} = \frac{15}{4y^2}$$

When we add or subtract with fractions we need to have the same denominator in each fraction.

To do this we need to find the lowest common denominator

$$\frac{x}{3} + \frac{2x+1}{2} = \frac{2x + 3(2x+1)}{3 \times 2}$$

LCM = 6

$$= \frac{2x + 6x + 3}{6}$$

$$= \frac{8x+3}{6}$$

Step 1: Find the lowest common denominator

Step 2: Cross multiply

Step 3: Simplify

Solve $\frac{x+5}{7} = 5$

$\times 7 \quad \times 7$

$x + 5 = 35$

$-5 \quad -5$

$x = 30$

Solve the equation $\frac{x+4}{2} = \frac{x+10}{3}$ ($\times 3$, $\times 2$)

$3(x+4) = 2(x+10)$

$3x + 12 = 2x + 20$

$-12 \quad -12$

$3x = 2x + 8$

$-x \quad -x$

$x = 8$

Solve

$\frac{2x+1}{3} + \frac{x-5}{2} = 4$ LCD = 6

$\frac{2(2x+1) + 3(x-5)}{3 \times 2} = 4$

$\frac{4x + 4 + 3x - 15}{6} = 4$

$\frac{7x - 11}{6} = 4$

$\times 6 \quad \times 6$

$7x - 11 = 24$

$+11 \quad +11$

$7x = 35$

$\div 7 \quad \div 7$

$x = 5$

Step 1: Find the lowest common denominator

Step 2: Cross Multiply

Step 3: Simplify the numerator

Step 4: Solve

Changing the subject

1. Circle the subject that we want to isolate
2. Identify what needs to be eliminated
3. Inverse the operation, and apply to both sides

When we have two terms on one side:

1) Factorise out the subject
2) Circle the subject that we want to isolate
3) Identify what needs to be eliminated
4) Inverse the operation, and apply both sides

Make x the subject of:

$$ax + b = cx + d$$

$$ax - cx = d - b$$

$$x(a - c) = d - b$$

$$x = \frac{d-b}{a-c}$$

Make c the subject

$cm + ca = z$

$c(m + a) = z$

$\div m + a \quad \div m + a$

$c = \frac{z}{m+a}$

Make x the subject of:

$q = \frac{2x+1}{2x-1}$

$q(2x-1) = 2x + 1$

$2xq - q = 2x + 1$

$2xq - 2x = 1 + q$

$x(2q - 2) = 1 + q$

$x = \frac{1+q}{2q-2}$

Surds

Simplifying Surds

Simplify:

$\sqrt{60}$

$\sqrt{4 \times 15}$

$\sqrt{4}\sqrt{15}$

$2\sqrt{15}$

1) Find two factors of the number (one of them needs to be a square number)

2) Chop the surd in half

3) Simplify

$\sqrt{\frac{25}{4}} = \frac{\sqrt{25}}{\sqrt{4}} = \frac{5}{2}$

Simplify $\sqrt{2} + \sqrt{8}$

When the numbers under the square root are different we have to simplify first

$= \sqrt{2} + \sqrt{4}\sqrt{2}$

$= \sqrt{2} + 2\sqrt{2}$

$= 3\sqrt{2}$

Multiplying Surds

$$\sqrt{5} \times \sqrt{6} = \sqrt{30}$$

$(2 + \sqrt{3})(2 + \sqrt{3})$

x	2	$\sqrt{3}$
2	4	$2\sqrt{3}$
$\sqrt{3}$	$2\sqrt{3}$	$\sqrt{9}$

$= 4 + 2\sqrt{3} + 2\sqrt{3} + \sqrt{9}$

$= 4 + 4\sqrt{3} + 3$

$= 7 + 4\sqrt{3}$

Rationalising the denominator

Rationalise the denominator for $\frac{3}{\sqrt{5}}$

$$\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$$

Rationalise the denominator for $\frac{2}{\sqrt{3}+1}$

$= \frac{2}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{2\sqrt{3}-2}{3 - \sqrt{3} + \sqrt{3} - 1}$

$= \frac{2\sqrt{3}-2}{2}$

$= \sqrt{3} - 1$

Algebraic Proof

SHOW...

"SHOW THAT THE SUM OF TWO CONSECUTIVE NUMBERS IS ALWAYS AN ODD NUMBER"

1 + 2 = 3 ✓
2 + 3 = 5 ✓
5 + 6 = 11 ✓
101 + 102 = 203 ✓

We have SHOWN that this works. We have not proved it works.

There could be, at some point, two consecutive numbers in fact give us an EVEN number

PROVE... ← Algebra is often used to establish the truth (to prove something)

"PROVE THAT THE SUM OF TWO CONSECUTIVE NUMBERS IS ALWAYS AN ODD NUMBER"

We can give any number, any letter. In this case, lets pick the letter "n"

Therefore the number directly after n is "n+1"

..."SUM OF TWO CONSECUTIVE NUMBERS..."

n + n + 1 = 2n + 1

"2n" is the nth term for the multiples of 2.

2n + 1 is one more than the multiples of 2, the odd numbers.

This PROVES that the statement is always true for any value of n

Identity:

An identity in maths is represented by the ≡ symbol

An identity is an equation which is always true no matter which values are chosen

$$(x+1)^2 \equiv x^2 + 2x + 1$$

Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4 for all positive integer values of n"

*Before you begin, highlight the key words and information from the question

Prove that $(3n + 1)^2 - (3n - 1)^2$ is a multiple of 4 for all positive integer values of n"

STEP 1: EXPAND

$(3n + 1)^2 - (3n - 1)^2$
$= (3n + 1)(3n + 1) - (3n - 1)(3n - 1)$
$= (9n^2 + 6n + 1) - (9n^2 - 6n + 1)$

STEP 2: SIMPLIFY

$= (9n^2 + 6n + 1) - (9n^2 - 6n + 1)$
$= 12n$

STEP 3: FACTORISE

$12n = 4 \times 3n$

STEP 3: JUSTIFY

$4 \times 3n$ → always divisible by 4 and hence a multiple of 4

Prove that
$(n + 1)^2 - (n - 1)^2 + 1$
is always odd for all positive integer values of n

Expand

$= (n+1)(n+1) - (n-1)(n-1) + 1$

$= (n^2 + 2n + 1) - (n^2 - 2n + 1) + 1$

Simplify

$= 4n + 1$

Justify $\quad 4 \times n$ is always even

Any even number add 1 is odd

Functions

Functions

A function is something which provides a rule on **how to map inputs to outputs**.
From primary school you might have seen this as a 'function machine'.

Name of the function (usually f or g) → $f(x) = 2x$ ← Input, Output

If $p = 3x$ and $x = \frac{y}{2}$
Write p in terms of y

$p = 3x$

$p = 3\left(\frac{y}{2}\right)$

$p = \frac{3y}{2}$

You are given that $f(x) = 3x + 1$

Find $f(1)$

This means we need to substitute x for 1

Find $f(1) = 3(1) + 1 = 3 + 1 = 4$

$f(x) = x^2 + 2$

If $f(a) = 38$, what is a?

$x^2 + 2 = 38$

$x^2 = 36$

$x = \pm 6$

Anything inside the bracket applies just to the x

Anything outside the bracket applies just to the whole function

If $f(x) = x + 1$ what is:

$f(x + 1) = (x + 1) + 1 = x + 2$

$f(x - 1) = (x - 1) + 1 = x$

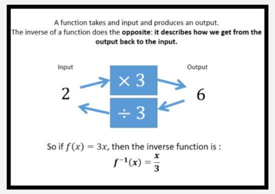
$f(x^2) = x^2 + 1$

$f(x)^2 = (x + 1)^2 = x^2 + 2x + 1$

$f(2x) = 2x + 1$

$2f(x) = 2(x + 1) = 2x + 2$

Inverse Functions



$f(x) = 2x + 1$

find $f^{-1}(x)$

$y = 2x + 1$

$y - 1 = 2x$

$x = \frac{y-1}{2}$ $\quad f^{-1}(x) = \frac{x-1}{2}$

STEP 1: Write the output $f(x)$ as y

STEP 2: Get the input in terms of the output (make x the subject).

STEP 3: Swap y back for x and x back for $f^{-1}(x)$.

Composite Functions

Given $f(x) = 2x$
$g(x) = x + 1$
find $fg(2)$

$g(2) = 2 + 1 = 3$

$f(3) = 2 \times 3 = 6$

Given $f(x) = 2x$
$g(x) = x + 1$
find $fg(1)$

$g(1) = 1 + 1 = 2$

$f(2) = 2 \times 2 = 4$

$f(x) = x + 1$
$g(x) = 2x$

Find $gf(x)$

$f(x) = x + 1$

$g(x + 1) = 2(x + 1) = 2x + 2$

$gf(x) = 2x + 2$