# Unit 18: Vectors

## Vector Basics

What is a vector?

A **vector** describes *direction* and *length*

The magnitude of a vector is its size

$$\begin{pmatrix} x \\ y \end{pmatrix}$$

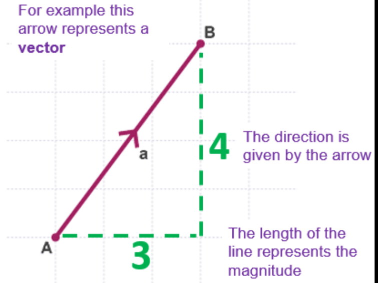
**X** = number of moves to the **right** or **left**

**Y** = number of moves **up** or **down**

This vector can be written in 3 ways

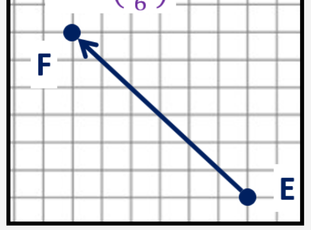
**a** $\quad \bar{a} \quad \overrightarrow{AB}$

For example this arrow represents a vector

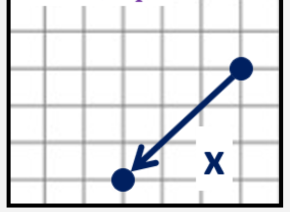



$\mathbf{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \bar{a} = \begin{pmatrix} 3 \\ 4 \end{pmatrix} \quad \overrightarrow{AB} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$

$\overrightarrow{EF} = \begin{pmatrix} -6 \\ 6 \end{pmatrix}$




$\mathbf{x} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$

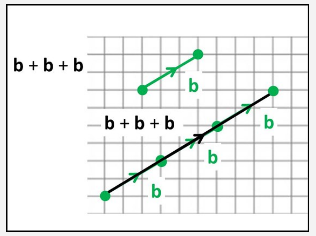



## Vector Arithmetic

What's another way of saying $b + b + b$?

**3b**

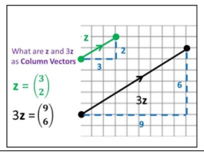
Scalar $\quad$ Vector




Multiplying a **Vector** by a Scalar

$\mathbf{z} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$

$3\mathbf{z} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$



To multiply a **Vector** by a Scalar. Write the **Vector** as a **Column Vector** then multiply each entry in the **Column Vector** by the Scalar

$3\mathbf{z} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \begin{matrix} \times 3 \\ \times 3 \end{matrix} = \begin{pmatrix} 9 \\ 6 \end{pmatrix}$

We can multiply a vector by a scalar

A scalar is a quantity that has size but no direction

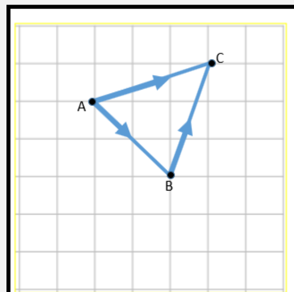
Vectors that have been multiplied by a scalar are **parallel**

If we add two or more vectors together we get a resultant vector

A resultant vector is the vector sum of two or more vectors

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} + \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 4 \\ -4 \end{pmatrix}$$

$$\begin{pmatrix} 5 \\ -2 \end{pmatrix} - \begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} 6 \\ 0 \end{pmatrix}$$




$\overrightarrow{AB} = \begin{pmatrix} 2 \\ -2 \end{pmatrix}$

$\overrightarrow{BC} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$

$\overrightarrow{AB} + \overrightarrow{BC} = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$

$\overrightarrow{BC} - \overrightarrow{AB} = \begin{pmatrix} -1 \\ 5 \end{pmatrix}$

## Midpoints and Ratio

### Midpoints of Vectors

3. $P$ is the point (1,5), $Q$ is the point (9,3)

a) Write down the vector $\overrightarrow{PQ}$

Write your answer as a column vector $\quad \begin{pmatrix} 8 \\ -2 \end{pmatrix}$

M is the midpoint of PQ

$\overrightarrow{PM} = \frac{1}{2}\overrightarrow{PQ} = \begin{pmatrix} 4 \\ -1 \end{pmatrix}$

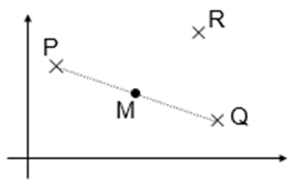
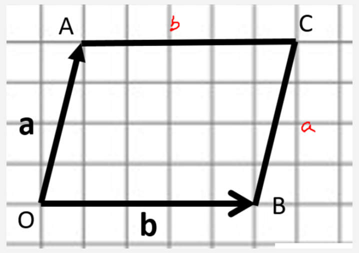



Diagram **NOT** accurately drawn

### Vectors in quadrilaterals

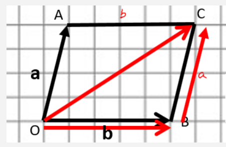



OACB is a parallelogram

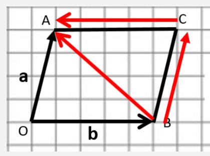
$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$

Find i) $\overrightarrow{OC}$ ii) $\overrightarrow{BA}$ iii) $\overrightarrow{CA}$

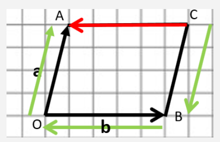
In terms of **a** and **b**




$\overrightarrow{OC} = \boldsymbol{b} + \boldsymbol{a}$




$\overrightarrow{BA} = \boldsymbol{a} - \boldsymbol{b}$




$\overrightarrow{CA} = -\boldsymbol{b}$

$\overrightarrow{CA} = -\cancel{\boldsymbol{a}} - \boldsymbol{b} + \cancel{\boldsymbol{a}}$

$\overrightarrow{CA} = -\boldsymbol{b}$

### Vectors with ratio

$\overrightarrow{AL} : \overrightarrow{LB} = 2:1$

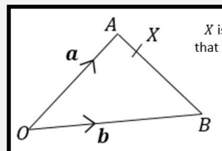
What is:


$\overrightarrow{AB} = a$

$\overrightarrow{AL} = \frac{2}{3}a$

There are 3 parts to the ratio So we are dealing with thirds

$\overrightarrow{LB} = \frac{1}{3}a$




$X$ is a point such that $AX:XB = 1:4$

a. $\overrightarrow{AB} = -\boldsymbol{a} + \boldsymbol{b}$

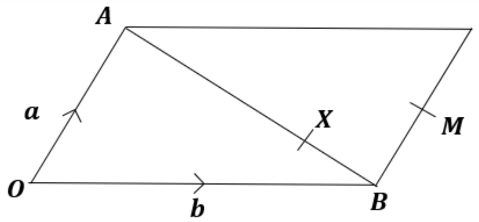
b. $\overrightarrow{AX} = -\frac{1}{5}\boldsymbol{a} + \frac{1}{5}\boldsymbol{b}$

c. $\overrightarrow{OX} = \frac{4}{5}\boldsymbol{a} + \frac{1}{5}\boldsymbol{b}$

d. $\overrightarrow{BX} = \frac{4}{5}\boldsymbol{a} - \frac{4}{5}\boldsymbol{b}$

## Vector Proof

How to <u>show</u> two vectors are parallel




$X$ is a point on $AB$ such that $AX:XB = 3:1$. $M$ is the midpoint of $BC$. Show that $\overrightarrow{XM}$ is parallel to $\overrightarrow{OC}$.

$\overrightarrow{OC} = a + b$

$\overrightarrow{XM} = \frac{1}{4}(-a + b) + \frac{1}{2}a = \frac{1}{4}a + \frac{1}{4}b$

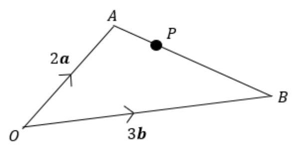
$= \frac{1}{4}(a + b)$

$\overrightarrow{XM}$ **<u>is a multiple of</u>** $\overrightarrow{OC}$ ∴ parallel.

For any proof question always find the vectors involved first, in this case $\overrightarrow{XM}$ and $\overrightarrow{OC}$.

The key is to factor out a scalar such that we see the same vector.

The magic words here are *"is a multiple of"*.




a) Find $\overrightarrow{AB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

$-2\boldsymbol{a} + 3\boldsymbol{b}$

b) $P$ is the point on $AB$ such that $AP:PB = 2:3$. Show that $\overrightarrow{OP}$ is parallel to the vector $\boldsymbol{a} + \boldsymbol{b}$.

M1 for $2\mathbf{a} \pm \frac{2}{5}('3\mathbf{b} - 2\mathbf{a}')$ OR $3\mathbf{b} \pm \frac{3}{5}('2\mathbf{a} - 3\mathbf{b}')$

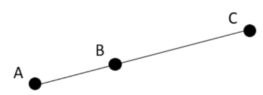
A1 for $\frac{6}{5}\mathbf{a} + \frac{6}{5}\mathbf{b}$ oe

A1 for $\frac{6}{5}(\mathbf{a} + \mathbf{b})$ is parallel to $\mathbf{a} + \mathbf{b}$ oe

### Collinear Points

Points A, B and C form a straight line if: $\overrightarrow{AB}$ and $\overrightarrow{BC}$ **are parallel (and B is a common point).**

**Alternatively, we could show $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel. This tends to be easier.**



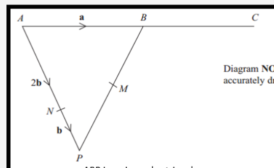




Diagram **NOT** accurately drawn

ABP is an isosceles triangle

$\overrightarrow{AN} = 2\boldsymbol{b}, \quad \overrightarrow{NP} = \boldsymbol{b}$

$B$ is the midpoint of $AC$. $M$ is the midpoint of $PB$.

a) Find $\overrightarrow{PB}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.

b) Show that $NMC$ is a straight line.

a $\quad -3\boldsymbol{b} + \boldsymbol{a}$

b $\quad \overrightarrow{NM} = b + \frac{1}{2}(a - 3b)$

$= b + \frac{1}{2}a - \frac{3}{2}b$

$= \frac{1}{2}a - \frac{1}{2}b$

$= \frac{\mathbf{1}}{\mathbf{2}}(\boldsymbol{a} - \boldsymbol{b})$

$\overrightarrow{NC} = -2b + 2a$

$= \mathbf{2}(\boldsymbol{a} - \boldsymbol{b})$

**$\overrightarrow{NC}$ is a multiple of $\overrightarrow{NM}$ ∴ $\overrightarrow{NC}$ is parallel to $\overrightarrow{NM}$.**

**$N$ is a common point.**

**∴ $NMC$ is a straight line.**