Unit 9: Equations and Inequalities

Solving Quadratic Equations

Solving by factorising

Solve the equation: $12x^2 - 14x = 0$

$2x(6x - 7) = 0$

Set each part equal to 0

$2x = 0$ $6x - 7 = 0$
$x = 0$ $+7 \; +7$
        $6x = 7$
        $\div 6 \; \div 6$
        $x = \frac{7}{6}$

Solve $x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

$x - 3 = 0$ $x - 4 = 0$
$+3 \; +3$ $+4 \; +4$
$x = 3$ $x = 4$

Solve: $4x^2 - 9 = 0$

$(2x + 3)(2x - 3) = 0$

$2x + 3 = 0$ $2x - 3 = 0$

$x = -\frac{3}{2}$ $x = \frac{3}{2}$

Steps:

Factorise the equation

Set each bracket equal to 0

Solve for x

Solving by using the quadratic formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Solve the following quadratic equation, giving your answer to two decimal places.

$4x^2 - 10x - 7 = 0$

$a = 4$ $b = -10$ $c = -7$

$x = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(4)(-7)}}{2(4)}$

$x = \frac{10 + \sqrt{212}}{8}$ $x = \frac{10 - \sqrt{212}}{8}$

$x_+ = 3.07 \; \textbf{(2dp)}$ $x_- = -0.57 \; \textbf{(2dp)}$

1) Compare with $ax^2 + bx + c$ to identify a, b and c

2) Substitute a, b and c into the **quadratic formula.**

3) Simplify the three sections.

4) Split into two and solve.

Solving by Elimination

Steps:

1) Make the x's or the y's the same using LCM

2) Label the equations 1 and 2

3) If the signs are the same subtract if not add

4) Isolate the variable by dividing

5) Find the unknown variable by substituting the known variable into one of the 2 equations.

$2x + 5y = 24$
$4x + 3y = 20$

    x2

LCM of 2 and 4 is 4

$4x + 10y = 48$ ①
$4x + 3y = 20$ ②

$7y = 28$
$\div 7$ $\div 7$
$y = 4$

$4x + 3(4) = 20$
$4x + 12 = 20$
$-12 \; -12$
$4x = 8$
$\div 4 \; \div 4$
$x = 2$

$x = 2, y = 4$

Simultaneous Equations

$3x - 2y = 0$ ①
$2x + y = 7$ ②

$y = 7 - 2x$ ②

$3x - 2(7 - 2x) = 0$

$3x - 14 + 4x = 0$

$7x - 14 = 0$

$7x = 14$

$x = 2$

$2(2) + y = 7$

$4 + y = 7$

$y = 3$

$x = 2, y = 3$

Solving by Substitution

Step 1: Rearrange the equation for x or y

Step 2: Substitute the equation into the other equation

Step 3: Solve the equation for x or y

Step 4: Substitute the value into the first equation

Completing the Square

Completing the square

Express the following quadratic expression in the form $(x + p)^2 + q$.

$x^2 - 4x + 10$

$\frac{4}{2} = 2$

$(x - 2)^2$

$(x - 2)^2 - 2^2$

$(x - 2)^2 - 4$

$(x - 2)^2 - 4 + 10$

$(x - 2)^2 + 6$

1) Halve the coefficient of x

2) Write the halved coefficient inside a bracket and square it

3) Square the halved coefficient and subtract it outside the bracket

4) Add the coefficient to the end and simplify

Turning Points

$(x - 2)^2 + 6$

The completed square form shows the turning point.

The minimum point of this equation is (2, 6)

Solving by completing the square

Solve the equation by completing the square

$x^2 - 8x - 2 = 0$

$(x - 4)^2 - 18 = 0$
$+18 \; +18$
$(x - 4)^2 = 18$
$\sqrt{} \;\;\;\;\; \sqrt{}$
$x - 4 = \pm\sqrt{18}$
$+4 \;\;\;\; +4$
$x = 4 \pm \sqrt{18}$

$x = 4 + \sqrt{18}$ or $x = 4 - \sqrt{18}$

Steps:

1) Get the equation into completed square form

2) Isolate the bracket on one side

3) Square root both sides

4) Isolate the x on one side

Inequalities

Solving linear inequalities

$4x + 7 > 35$
$-7 \;\;\;\;\; -7$
$4x > 28$
$\div 4 \;\;\;\;\; \div 4$
$x > 7$

$-4x + 7 > 35$
$-7 \;\;\;\;\; -7$
$-4x > 28$
$\div -4 \;\;\;\; \div -4$
$x < -7$

Inequalities on a number line

$x > 3$ — This is known as a 'strict' inequality.

Means: x is (strictly) greater than 3.

$x < -1$

Means: x is (strictly) less than -1.

$x \geq 4$

Means: x is greater than or equal to 4.

$x \leq 5$

Means: x is less than or equal to 5.
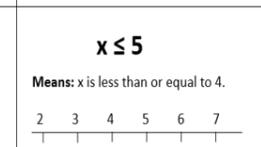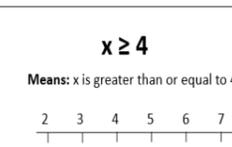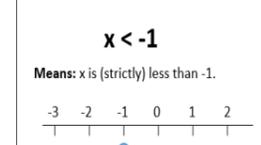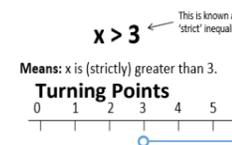
Solving Quadratic inequalities

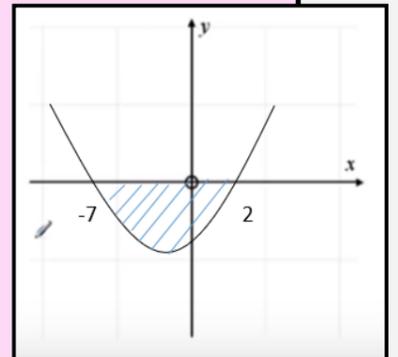Solve $x^2 + 5x - 14 < 0$

$(x + 7)(x - 2) < 0$

$(x + 7) > 0$ $(x - 2) < 0$
$x > -7$ $x < 2$

$-7 < x < 2$

Steps:

1) Factorise

2) Sketch the curve

3) Shade the area that satisfies the inequality

4) Write the inequality